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Effect of Damping on Shock Spectra of Impulse Loads

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Abstract–In the present paper, single degree of freedom systems have been considered for studying the effects of damping on various response quantities and shock spectra for three different impulse excitations—rectangular, half-sine and two-legged triangular force pulses. The Duhamel's integral method based upon linear force variation over time step has been explained in this paper, which is also the methodology to estimate the response of the systems in this study. Numerical solution of the problem has been carried out. Responses of ten systems with different time periods have been plotted for different frequency ratios. Damping ratios considered for shock spectra are 0, 0.02, 0.05, 0.1 and 0.2. It has been observed that, as the duration of the impulse load approaches time period of the system, amplitude vibration becomes maximum. Further, the dynamic magnification factor (DMF) in the shock spectra is maximum for rectangular pulse followed by half-sine wave and triangular pulse, for a given magnitude. Similar trend is seen for damped systems.

Index terms – Impulse loads, Dynamic magnification factor, shock spectra

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1. INTRODUCTION

Due to increasing number of terrorist attacks on structures of importance and accidents in laboratories, it is becoming increasingly important to analyse and design structures to be safer against blast loads. Generally, it has been accepted that single degree of freedom analysis are best suited for such problems. Therefore, in this paper the effect of damping on shock spectra has been investigated along with effect of blast load duration on a structural system with different time periods. The equation of motion of a damped single degree of freedom structure having mass m, damping coefficient c and stiffness k, subjected to a dynamic force $F(t)$ is given by

$$m\ddot{y}(t) + c\dot{y}(t) + ky(t) = F(t) \tag{1}$$

where, y is the displacement of the structure at time t. There are several methods available to solve Equation (1) depending upon the nature of the force applied on the structure. In case, $F(t)$ is a simple well defined pulse like rectangular, triangular, step force or half-sine pulse, closed-form solutions of response can readily be derived as explained briefly in the following section. Analytical solutions are also possible if the force $F(t)$ is a periodic force that can be expressed as a function of time, like harmonic excitations. For harmonic excitation of frequency $\overline{\omega}$ and amplitude F_0, the complete solution is obtained as a sum of the complimentary solution and the particular solution, as given by the following equation.

$$y(t) = e^{-\xi\omega t}\left(A\cos\omega_d t + B\sin\omega_d t\right) + y_{st} D\sin\left(\overline{\omega}t - \theta\right) \tag{2}$$

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where, D is dynamic magnification factor given by $D = 1/\sqrt{\left(1-\beta^2\right)^2 + \left(2\xi\beta\right)^2}$, $\beta = \bar{\omega}/\omega_d$ being the frequency ratio, and y_{st} is the static displacement given by F_0/k. The first part of Equation (2) is transient response, and dies out after some time due to the term $e^{-\xi\omega t}$. Thereafter, the response primarily consists of the second part, known as steady state response.

In some cases, the force is periodic but not harmonic such as wave loading on offshore structures, wind forces due to vortex shedding on tall and slender structures, etc. In such cases, analysis can be carried out utilizing discrete Fourier transform technique. In practice, the excitation consists of arbitrarily varying force for example, wind force or earthquake acceleration. Obtaining a closed form solution of the structural response for such forces is quite tedious or impossible. Analytical solution of the equation of motion for a single degree of freedom (SDOF) system is usually not possible if the system is non-linear [Chopra, 2001]. In such cases, time-stepping numerical techniques are used to solve the equation of motion such as central-difference technique, Newmark's methods, Wilson-θ, etc. The selection of the method depends upon a number of factors such as their accuracy, convergence, stability properties and feasibility of computer implementation. Apart from these techniques, Duhamel's method can also be used to obtain the response directly using convolution integral. This gives response of the system due to any impulse force $F(t)$, as follows.

$$y(t) = \frac{1}{m\omega}\int_0^t F(\tau)\sin\omega(t-\tau)d\tau \tag{3}$$

The integral in Equation (3) is known as Duhamel's integral. It can be evaluated to get closed form solutions for simple pulses whereas for arbitrarily varying loads, numerical integration needs to be done over a suitably chosen small time step. Size of the time step is very crucial for convergence of the results. In order to include the effect of initial conditions, Equation (3) is to be added to the free-vibration solution. Therefore, total response becomes,

$$y(t) = y_0\cos\omega t + \frac{\dot{y}_0}{\omega}\sin\omega t + \frac{1}{m\omega}\int_0^t F(\tau)\sin\omega(t-\tau)d\tau \tag{4}$$

Numerical solution considering the force to be linear during the time step has been found to give best solutions. The procedure to obtain the analytical solutions for some typical pulse forces is explained further in this text.

2. DMF AND SHOCK SPECTRA

Shock spectra for a given impulse is the plot of dynamic magnification factors of a number of SDOF systems subjected to this impulse versus frequency or time period of the SDOF systems. Dynamic magnification factor is obtained by dividing maximum displacement by static displacement, i.e., DMF= $\left[y(t)\right]_{max}/y_{st}$. Few cases of impulse loads are presented below.

2.1 Constant Step Force

Let a force of magnitude F_0 be applied to the structure suddenly and this force continues to act. The definition of the force is, $F(t) = F_0$ for all $t \geq 0$, as given in Figure 1.

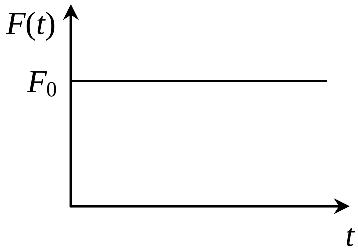

Figure 1: Constant force.

Using the Equation (3), and putting $F(t) = F_0$, response at any time t is derived to be,

$$y(t) = y_{st}(1 - \cos\omega t) \tag{5}$$

2.2 Rectangular pulse

Let t_d be the duration of the rectangular force of magnitude F_0. The definition of the pulse is given as

$$\begin{aligned} F(t) &= F_0 \text{ for } 0 \le t \le t_d \\ &= 0 \text{ for } t > t_d \end{aligned} \tag{6}$$

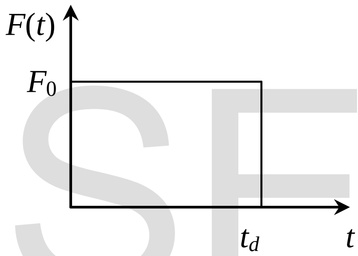

Figure 2: Rectangular pulse force.

Response of the system during the pulse is the same as for constant force given by Equation (6). After time t_d, total response will be given by Equation (4). Here, y_0 and $\dot{y}_0$ are in fact the displacement and velocity at the end of pulse, i.e., at time t_d. And, they act as initial conditions for second phase of vibration starting after the pulse vanishes. Equation (5) can be solved to get the desired response. Solution is thus given as follows.

$$y(t) = \begin{cases} y_{st}(1 - \cos\omega t) & \text{for } t \le t_d \\ y_{st}\left[\cos\omega(t - t_d) - \cos\omega t\right] & \text{for } t \ge t_d \end{cases} \tag{7}$$

2.3 Half sine-wave force

Force definition,

$$\begin{aligned} F(t) &= F_0 \sin\overline{\omega}t && \text{for } 0 \le t \le t_d \\ &= 0 && \text{for } \frac{t_d}{2} \le t \le t_d \end{aligned} \tag{8}$$

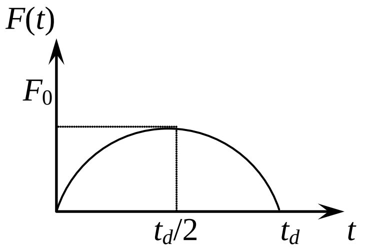

Figure 3: Half-sine wave pulse.

Solving Equation (3) for first phase of vibration ($0 \le t \le t_d$) and Equation (4) for second phase of vibration ($t \ge t_d$), we get the following solutions.

$$y(t) = \frac{y_{st}}{1-\beta^2}\left[\sin\bar{\omega}t - \beta\sin\omega t\right] \qquad \text{for } 0 \le t \le t_d$$
$$= \frac{2\beta y_{st}}{1-\beta^2}\cos\left(\frac{\pi}{2\beta}\right) \qquad \text{for } t > t_d \text{ and } \beta > 1 \tag{9}$$

DMFs obtained for two special cases as given below.

Case-1: $\frac{t_d}{T} \ne \frac{1}{2}$

$$\frac{y(t)}{y_{st}} = \frac{1}{1-(T/2t_d)^2}\left[\sin\left(\pi\frac{t}{t_d}\right) - \frac{T}{t_d}\sin\left(2\pi\frac{t}{T}\right)\right] \qquad \text{for } 0 \le t \le t_d$$
$$= \frac{(T/2t_d)\cos(\pi t_d/T)}{(T/2t_d)^2-1}\sin\left[2\pi\left(\frac{t}{T}-\frac{t_d}{2T}\right)\right] \qquad \text{for } t > t_d \tag{10}$$

Case-2: $\frac{t_d}{T} = \frac{1}{2}$

$$\frac{y(t)}{y_{st}} = \frac{1}{2}\left[\sin\left(\frac{2\pi t}{T}\right) - \frac{2\pi t}{T}\cos\left(\frac{2\pi t}{T}\right)\right] \qquad \text{for } 0 \le t \le t_d$$
$$= \frac{\pi}{2}\cos 2\pi\left(\frac{t}{T}-\frac{1}{2}\right) \qquad \text{for } t > t_d \tag{11}$$

For double-triangular pulse, responses of SDOF system are also obtained in the similar manner as explained above.

3. RESPONSE OF UNDER-DAMPED SDOF SYSTEM

In case of viscously damped SDOF system, the solution can be evaluated by introducing the term $e^{-\xi\omega(t-\tau)}$ in the Duhamel's integral in Equations (3) and (4). Thus, the new equations are given below.

For forced vibration phase with zero initial conditions,

$$y(t) = \frac{1}{m\omega}\int_0^t e^{-\xi\omega(t-\tau)}F(\tau)\sin\omega(t-\tau)d\tau \tag{12}$$

For free vibration phase,

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$$y(t) = y_0 \cos \omega t + \frac{\dot{y}_0}{\omega} \sin \omega t + \frac{1}{m\omega} \int_0^t e^{-\xi\omega(t-\tau)} F(\tau) \sin \omega(t-\tau) d\tau \quad (13)$$

4. NUMERICAL STUDY

The program developed to solve response and DMF equations given above has been verified with a problem chosen from [2]. Table-1 gives force-definition and Table-2 shows the results which are found to be conforming to an acceptable level of accuracy. The responses of ten SDOF systems (for ten ratios t_d/T) are obtained by numerical simulation. These responses are shown in Figures 4 to 6. Further, the dynamic magnification factors are also computed for these cases and plotted as shock spectra in Figure 7 for no damping for equal-magnitude pulses. Shock spectra (DMF vs t_d/T) are also plotted for damped SDOF systems for different damping ratios, ξ = 0.02, 0.05, 0.1 and 0.2 as given in Figure 8.

Table-1: Force definition [2]

Time	0	0.02	0.04	0.06
Force	0	120000	120000	0

Mass = 100 kg, Stiffness = 10^5 N/m, Damping ratio = 0.05, Maximum time = 0.12 s, Time step = 0.005 s.

Table-2: Results of Program [2]

Time (s)	**Force**	**Disp.**	**Vel.**	**Acc.**	**Support Reaction**
0	0	0	0	0	0
0.005	30000	0.001244	0.744511	296.4021	266.2577
0.01	60000	0.009872	2.944021	580.8181	1356.947
0.015	90000	0.032982	6.521264	846.3962	3889.816
0.02	120000	0.077199	11.36608	1086.858	8515.612
0.025	120000	0.14728	16.59417	1000.244	15634.93
0.03	120000	0.242319	21.3295	890.2316	25153.09
0.035	120000	0.359571	25.46261	759.9097	36847.58
0.04	120000	0.495785	28.90063	612.823	50413.83
0.045	90000	0.64605	30.82488	156.4731	65336.24
0.05	60000	0.800234	30.47086	-296.591	80601.41
0.055	30000	0.947033	27.88308	-735.207	95112.86
0.06	0	1.075504	23.16063	-1148.74	107799.5
0.065	0	1.176573	17.1984	-1230.96	117782.9
0.07	0	1.246934	10.90402	-1281.42	124741.1
0.075	0	1.285328	4.438476	-1299.36	128540.4
0.08	0	1.291305	-2.0355	-1284.87	129132.1
0.085	0	1.265226	-8.35753	-1238.8	126550.2
0.09	0	1.20824	-14.3735	-1162.79	120909.5
0.095	0	1.122242	-19.9394	-1059.19	112401.2

0.1	0	1.009815	-24.9244	-930.997	101288.6

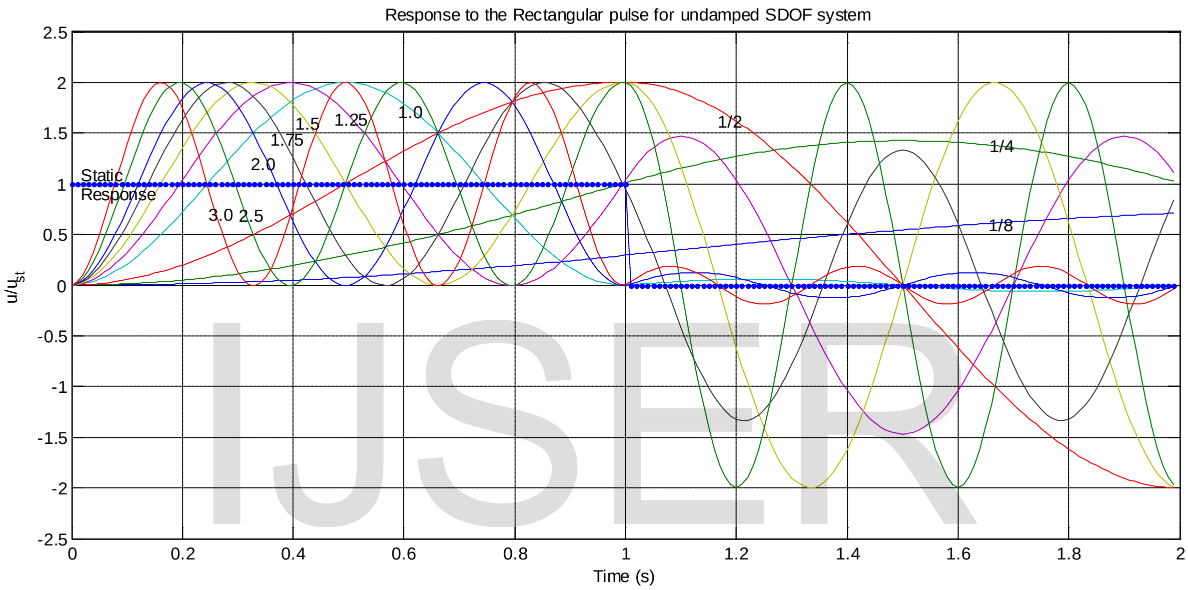

Figure 4: Response of undamped SDOF systems to rectangular pulse.

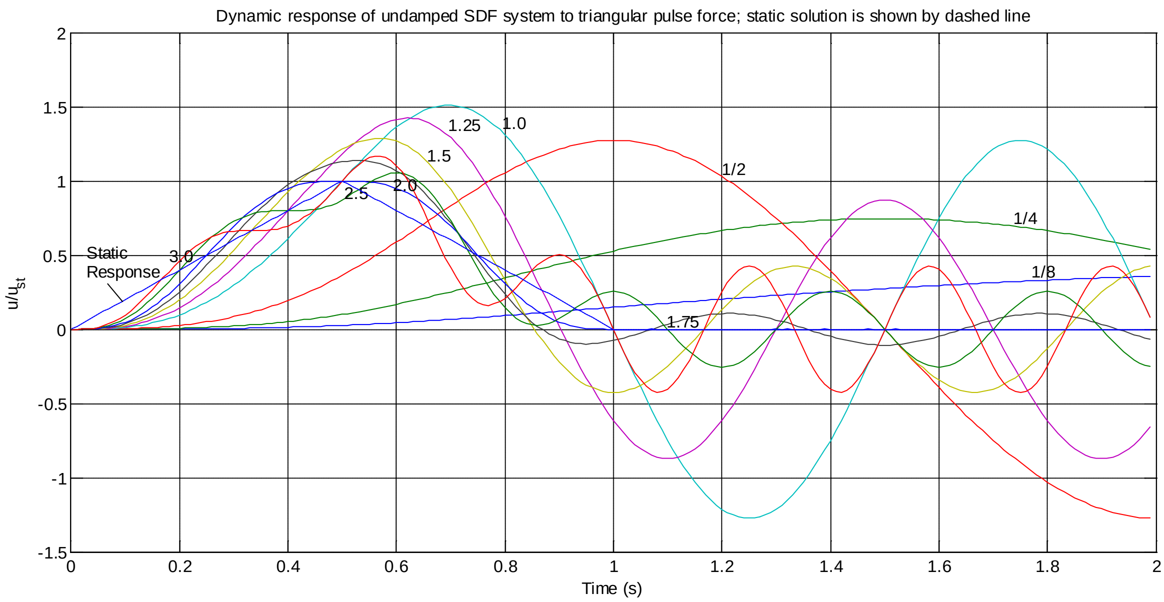

Figure 5: Response of undamped SDOF systems to triangular pulse.

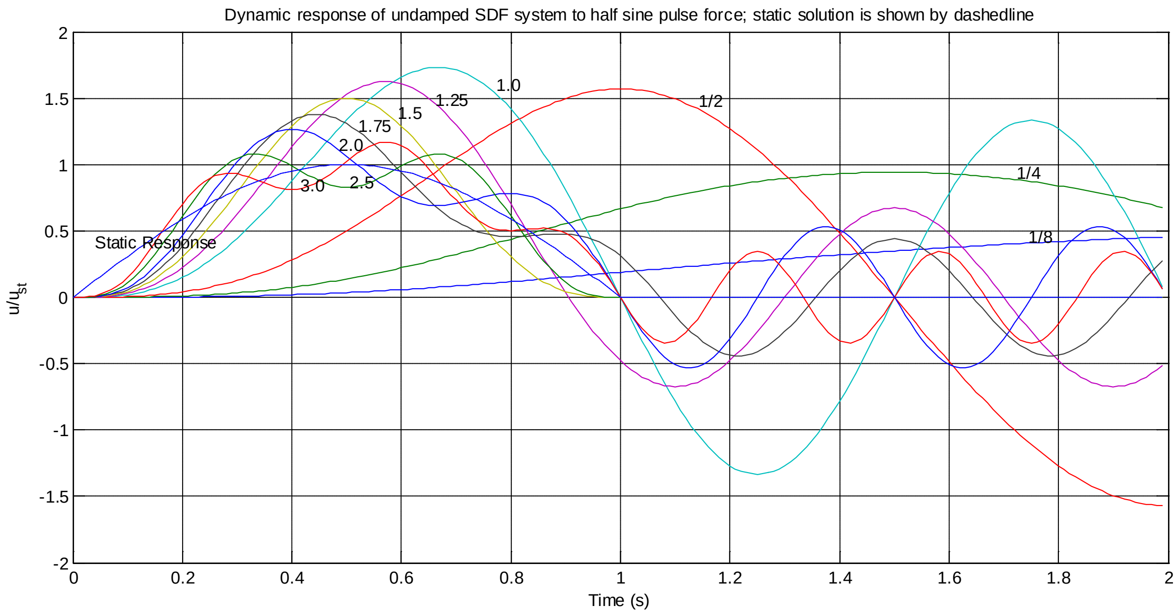

Figure 6: Response of undamped SDOF systems to half-sine pulse.

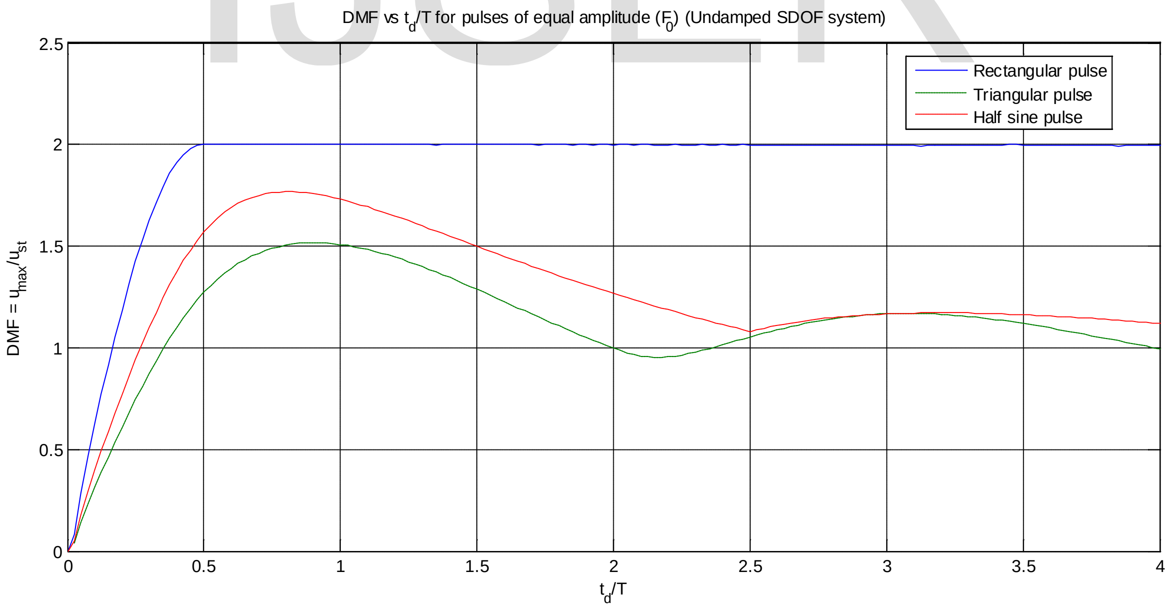

Figure 7: Shock spectra (DMF vs t_d/T) for various pulses of same amplitude ($\xi = 0$).

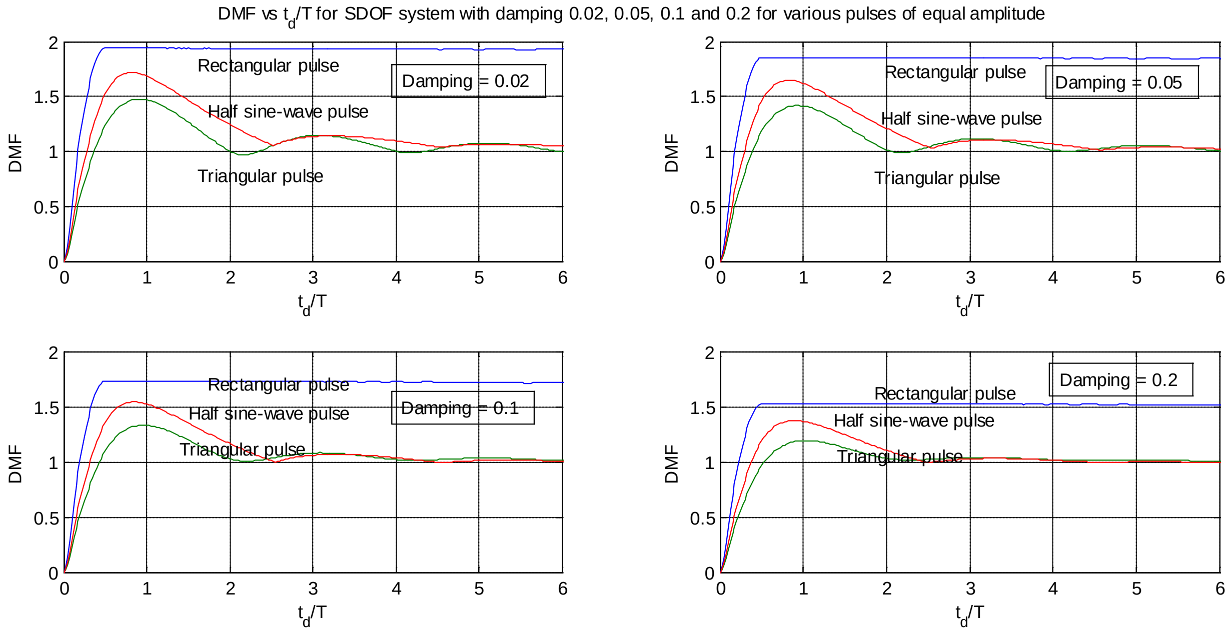

Figure 8: Shock spectra (DMF vs t_d/T) for various pulses of equal amplitude for damping ratios 0.02, 0.05, 0.1 and 0.2.

5. CONCLUSIONS

The following conclusions have been drawn from the study carried out in this paper.

(1) For a given value of amplitude of impulse load, dynamic magnification factor is the maximum for rectangular pulse followed by that for half-sine wave and triangular pulse. This can be attributed to the amount of impulse is area under force-time graph.

(2) DMF for rectangular pulse reaches maximum value and stays there for higher time period systems, especially beyond twice the pulse duration.

(3) For half-sine wave and triangular pulses, shock spectra reaches its peak for all damping ratios, before unity frequency ratio and decreases gradually to one.

(4) Increase in damping reduces the DMF values for all impulse loads. Reduction seems proportion for change in damping.

(5) As the duration of the impulse load approaches time period of the system, amplitude of vibration becomes maximum.

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